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MRS Criterion for Flow Separation over Moving Walls

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Steady, incompressible, laminar boundary-layer flows over moving walls are numerically investigated with special regard to the MRS criterion for flow separation. To analyze separated flows without meeting any singularity at the separation point, a set of approximate equations (instead of the boundary-layer equations) is solved under the condition of prescribed external velocity distributions. Three different types of flows are considered, one of which is quite similar to that treated by Tsahalis. It is found that, at least for the flows treated in the present paper, the MRS criterion is not satisfied for the case of upstream-moving walls, although it is satisfied for the case of downstream-moving walls.

Introduction

IT is now well known that the classical criterion for flow separation, $\partial u/\partial y=0$ at $y=0$, is inadequate for cases other than steady flow over fixed walls. Moore,[1] Rott,[2] and Sears[3] independently proposed a more appropriate criterion for the case of steady flows over moving walls, that is, $\partial u/\partial y=0$ at $u=0$. This criterion is called the "MRS criterion" after those authors. The streamlines and velocity profiles expected from the MRS criterion are depicted by Sears and Telionis,[4] and Williams.[5] A number of attempts have been made to verify the MRS model. They are also reviewed by Sears and Telionis, and Williams. The MRS model seems to have been strongly supported by experiments, at least for the case of downstream-moving walls. Ludwig[6] investigated separation on a rotating cylinder in steady flow by using hot-wire anemometers. He obtained velocity profiles that correspond to those expected by the MRS model. Koromilas and Telionis[7] obtained streamlines around a rotating cylinder, by the method of flow visualization, which are quite similar near a separation point to those suggested by the MRS model, both for the case of upstream- and downstream-moving walls. From the viewpoint of theoretical work, most of the investigations are confined to the case of downstream-moving walls. Telionis and Werle[8] and Tsahalis and Telionis[9] studied the case of flow over a parabola with a downstream-moving wall. They showed that integration of the boundary-layer equations through a point of vanishing wall shear presents no special difficulties and that the solution is terminated in the vicinity of a station where the velocity and shear simultaneously vanish in a singular fashion. This is taken as a verification of the MRS model for steady flow over downstream-moving walls. Danberg and Fansler[10] obtained a family of similar solutions of the boundary-layer equations whose velocity profiles correspond to the MRS model for the case of downstream-moving walls.

For the case of upstream-moving walls, the u component of velocity is negative near the wall and positive in the upper part of the boundary layer; therefore, it requires special techniques to obtain a numerical solution. Tsahalis[11] obtained a solution for the case of an upstream-moving wall. The solution is obtained as an asymptotic solution, for large time, of the unsteady boundary-layer equations. The velocity profile obtained is close to an MRS profile. As pointed out by Danberg and Fansler,[10] however, in a steady state the boundary-layer equations do not permit an MRS profile. Tsahalis thinks that the full Navier-Stokes equations may permit an MRS profile so that an approach to "near MRS-like" profiles may be a signal of incipient separation.[12] Further confirmation of his hypothesis is needed. Fansler and Danberg[12] solved the integral boundary-layer equations and obtained similarity solutions. An MRS profile was not found from their similarity solutions for upstream-moving walls, although they obtained an MRS profile for downstream-moving walls.

Most of the calculations in the previous theoretical works for the case of steady flow over moving walls are confined up to the MRS point (or to the point where the boundary-layer solution breaks down) and, to the author's knowledge, no descriptions have been presented until now for the case of upstream-moving walls. In the present paper, numerical calculations are performed through the regions of the separation bubble, for the cases of both upstream- and downstream-moving walls. Instead of the boundary-layer equations, approximate equations are numerically solved by finite-difference method. Three different types of flow are treated, one of which is quite similar to that treated by Tsahalis.[11] Flow properties over moving walls are discussed concerning the MRS criterion.

Governing Equations and Boundary Conditions

We consider two-dimensional, steady, incompressible, laminar viscous flows over a flat plate. The coordinate system (x,y) is fixed and the flat plate moves at $u=u_w$ either upstream ($u_w<0$) or downstream ($u_w>0$). As the governing equations, we adopt the following approximate equations:

$$u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}=\frac{1}{Re}\frac{\partial^2\omega}{\partial y^2} \tag{1a}$$

$$\omega=\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial x^2} \tag{1b}$$

$$u=\partial\psi/\partial y,\quad v=-\partial\psi/\partial x \tag{1c}$$

where $Re(=u_\infty L/\nu)$ is the Reynolds number and the variables are nondimensionalized by the upstream velocity u_∞ and the characteristic length L. This set of equations is essentially the same as the interacting boundary-layer model proposed by Werle and Bernstein,[13] and solutions of the equations have been numerically verified for the case of fixed walls to approximate the Navier-Stokes solutions closely.[14] The equations have the following advantages in comparison with the boundary-layer equations. First, the equations can be solved with a prescribed external velocity profile without meeting any singularity at separation point. Next, while the steady boundary-layer equations do not permit solutions with $u=\partial u/\partial y=0$ for the case of upstream moving walls,[10] Eqs. (1) may permit such solutions because they allow pressure variation in the y direction.[14]

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The boundary conditions are prescribed as,

$y=0$: $u=u_w$ and $v=0$

$y=y_e$: $\omega=0$ and $u=U_e(x)$ prescribed

$x=x_1$: ω and ψ given by boundary-layer solution

$x=x_2$: $\partial^2\psi/\partial x^2=0$ (2)

where x_1 is the upstream boundary, x_2 the downstream boundary, and y_e the boundary-layer edge. We introduce new variables,

$$\bar{u}=u-u_w(x), \qquad \bar{v}=v$$

$$\bar{\omega}=\partial\bar{u}/\partial y-\partial\bar{v}/\partial x=\omega \tag{3}$$

With Eqs. (3), Eqs. (1) and the boundary conditions [Eqs. (2)] are rewritten as follows:

$$(\bar{u}+u_w)\frac{\partial\bar{\omega}}{\partial x}+\bar{v}\frac{\partial\bar{\omega}}{\partial y}=\frac{1}{Re}\frac{\partial^2\bar{\omega}}{\partial y^2} \tag{4a}$$

$$\bar{\omega}=\frac{\partial^2\bar{\psi}}{\partial y^2}+\frac{\partial^2\bar{\psi}}{\partial x^2}+u_w''y \tag{4b}$$

$$\bar{u}=\partial\bar{\psi}/\partial y, \qquad \bar{v}=-\partial\bar{\psi}/\partial x-u_w'y \tag{4c}$$

$y=0$: $\bar{u}=0$ and $\bar{v}=0$

$y=y_e$: $\bar{\omega}=0$ and $\bar{u}=\bar{U}_e=U_e-u_w(x)$ prescribed

$x=x_1$: $\bar{\omega}$ and $\bar{\psi}$ given by the boundary-layer solution

$x=x_2$: $\partial^2\bar{\psi}/\partial x^2=-u_w''y$ (5)

where the prime denotes the derivative with respect to x.

Method of Solution

In order to solve Eqs. (4) with the boundary conditions of Eqs. (5), we adopted the following procedure. First, we substitute the solution $\bar{\omega}^{(0)}$ of Eqs. (4) for the case of a fixed wall into the term $u_w(\partial\bar{\omega}/\partial x)$ of Eq. (4a), and solve the equation

$$\bar{u}\frac{\partial\bar{\omega}^{(1)}}{\partial x}+\bar{v}\frac{\partial\bar{\omega}^{(1)}}{\partial y}=\frac{1}{Re}\frac{\partial^2\bar{\omega}^{(1)}}{\partial y^2}-u_w\frac{\partial\bar{\omega}^{(0)}}{\partial x} \tag{6}$$

together with Eqs. (4b) and (4c). After the set of solutions $\bar{\omega}^{(1)}$ and $\bar{\psi}^{(1)}$ is obtained, we substitute $\bar{\omega}^{(1)}$ into the term $u_w(\partial\bar{\omega}/\partial x)$, obtaining $\bar{\omega}^{(2)}$ and $\bar{\psi}^{(2)}$. Similarly, we continue to solve the equations,

$$\bar{u}\frac{\partial\bar{\omega}^{(K+1)}}{\partial x}+\bar{v}\frac{\partial\bar{\omega}^{(K+1)}}{\partial y}=\frac{1}{Re}\frac{\partial^2\bar{\omega}^{(K+1)}}{\partial y^2}-u_w\frac{\partial\bar{\omega}^{(K)}}{\partial x} \tag{7a}$$

$$\bar{\omega}^{(K+1)}=\frac{\partial^2\bar{\psi}^{(K+1)}}{\partial y^2}+\frac{\partial^2\bar{\psi}^{(K+1)}}{\partial x^2}+u_w''y \tag{7b}$$

$$\bar{u}=\partial\bar{\psi}^{(K+1)}/\partial y, \qquad \bar{v}=-\partial\bar{\psi}^{(K+1)}/\partial x-u_w'y \tag{7c}$$

until the maximum changes in $|\bar{\omega}^{(K+1)}-\bar{\omega}^{(K)}|$ and $|\bar{\psi}^{(K+1)}-\bar{\psi}^{(K)}|$ became less than certain small values.

At each iteration level $K+1$, a set of Eqs. (7) was solved by the finite-difference method. For $\bar{u}\geq 0$, a three-point backward scheme given by Carter and Wornom[15] was used for Eqs. (7a) and (7c). For $\bar{u}<0$, Eqs. (7a) and (7c) were approximated by central-difference expressions, but only for $\partial\bar{\omega}/\partial x$ was a first-order forward-difference expression used. Independent of whether $\bar{u}$ was positive or negative, Eq. (7b) was evaluated with central-difference expressions. Since the resulting matrix is tridiagonal, Eqs. (7) can be solved quickly with a simple algorithm. The set of Eqs. (7) was iteratively solved on each line of x, starting from upstream to downstream. Then, this iteration was repeated until the maximum changes in $\bar{\omega}^{(K+1)}$ and $\bar{\psi}^{(K+1)}$ between two successive iterations became less than certain small values.

In actual computations, convergence was assumed when $|\bar{\omega}^{(K+1)}-\bar{\omega}^{(K)}|<3\times10^{-5}$ and $|\bar{\psi}^{(K+1)}-\bar{\psi}^{(K)}|<3\times10^{-5}$.

Results and Discussion

The next three different types of flows are considered as follows,

Case I: $U_e(x)=1-x$ $\quad(x_1\leq x\leq x_3)$

$=1-x_3\equiv U_0$ (const) $\quad(x_3\leq x\leq x_2)$

$u_w(x)=\alpha$ (=const)

$x_1=0.05,\ x_2=0.45,\ y_e=0.05,\ \Delta x=0.005,\ \Delta y=0.001$

$U_0=0.76,\ 0.78,$ or 0.80

Case II: Flow conditions are the same as in case I, except the wall velocity;

$u_w=\alpha(x-x_1)$ $\quad(x_1\leq x\leq x_3)$

$=\alpha(x_3-x_1)$ (=const) $\quad(x_3\leq x\leq x_2)$

Case III: $U_e(x)=1.0$ $\quad(x_1\leq x\leq 0.0)$

$=1.0+Ax^2(3-2x)$ $\quad(0.0\leq x\leq 1.0)$

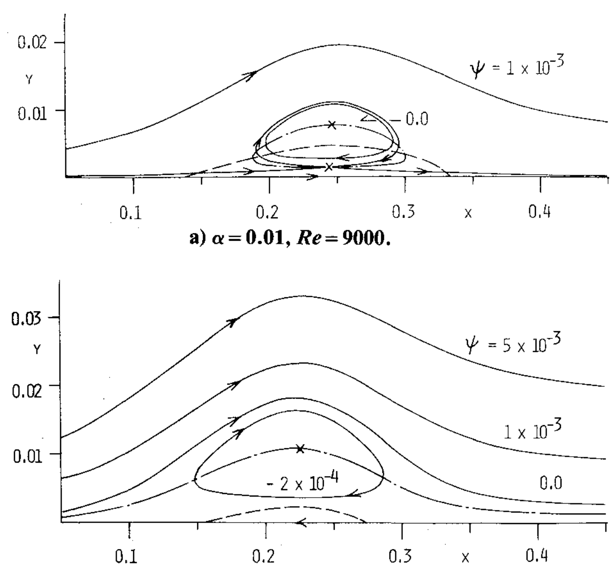
$=1.0+A$ (=const) $\quad(1.0\leq x\leq x_2)$

$u_w(x)=\alpha$ (=const)

$x_1=-2.0,\ x_2=5.0,\ y_e=0.2,\ \Delta x=0.05,\ \Delta y=0.005$

$A=-0.12,\ -0.1,\ -0.08,$ or -0.06

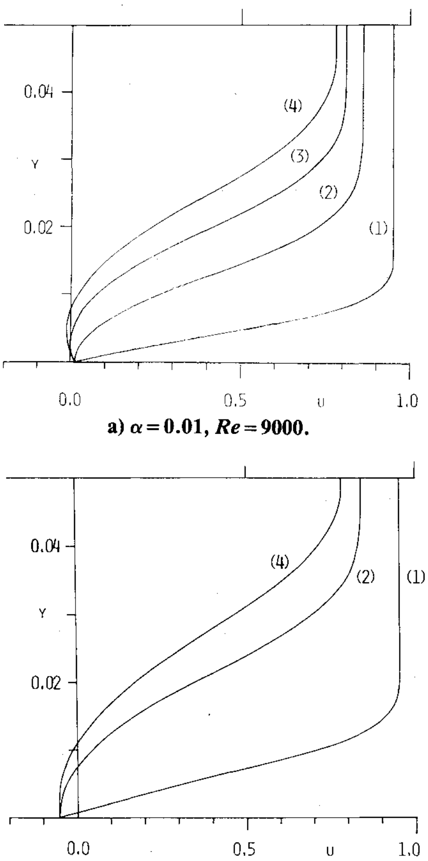
The Reynolds numbers used are $Re=3000$, 6000, 9000, and 12,000. As for the upstream condition at $x=x_1$, Howarth's boundary-layer solution was used for case II, while similarity



a) $\alpha=0.01$, $Re=9000$.

b) $\alpha=-0.05$, $Re=6000$.

Fig. 1 Typical streamlines for case I: $U_0=0.78$, $X=$ stagnation point (— — — $u=0$, ---- $\partial u/\partial y=0$).



a) $\alpha = 0.01$, $Re = 9000$.

b) $\alpha = -0.05$, $Re = 6000$.

Fig. 2 Typical velocity profiles for case I: $U_0 = 0.78$. Curves: 1) upstream boundary $x = x_1$, 2) at a station of zero wall shear, 3) at an MRS station, 4) at a center of bubbles (a stagnation point).

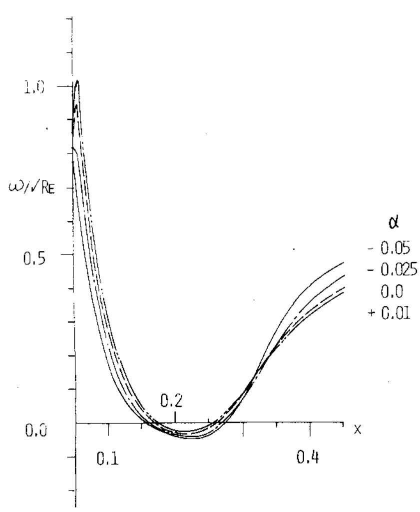


Fig. 3 Wall shear distributions for case I: $Re = 6000$, $U_0 = 0.78$.

solutions of the boundary-layer equations were used for cases I and III. For the case of a fixed wall ($\alpha = 0$) in case II, the problem has been solved by Briley[16] using the Navier-Stokes equations. The present approximate equations (1) have been verified to approximate the Briley's Navier-Stokes solutions closely.[14] Case II is quite similar to the problem treated by Tsahalis,[11] who solved the unsteady boundary-layer equations for the case of Howarth retarded flow, e.g., $U_e(x) = 1 - Ax$, $u_w(x) = -ACx$ ($A = 0.12$, $AC > 0$).

Typical streamlines and velocity profiles for case I obtained by the present method are shown in Figs. 1 and 2, respectively. In Fig. 1, the broken line denotes the $\partial u/\partial y = 0$ line and

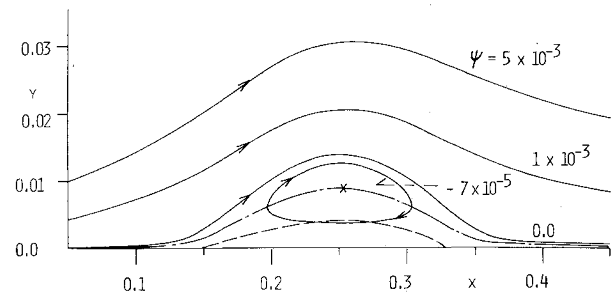


Fig. 4 Typical streamlines for case II: $Re = 9000$, $U_0 = 0.78$, $\alpha = -0.05$, X = stagnation point (— — — $u = 0$, ---- $\partial u/\partial y = 0$).

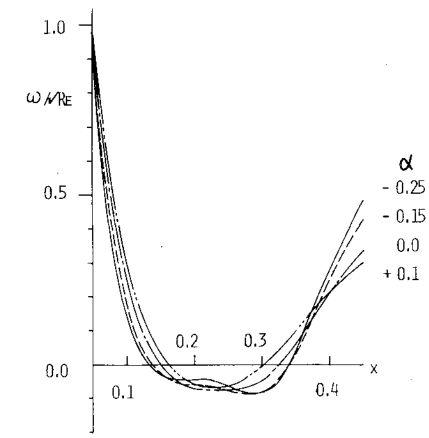


Fig. 5 Wall shear distributions for case II: $Re = 9000$, $U_0 = 0.78$.

the dashed line denotes the $u = 0$ line. The MRS point is that where the two lines cross. For the case of a downstream-moving wall, two MRS points exist near a recirculating region, as seen from Fig. 1a. The upstream MRS point corresponds to the so-called MRS separation point in the previous investigations. The velocity profile at the upstream MRS point coincides with that expected by the MRS model (Fig. 2a). For the case of an upstream-moving wall, an MRS point does not exist (Fig. 1b), namely, the $u = 0$ line and the $\partial u/\partial y = 0$ line do not cross. The velocity profiles expected by the MRS model do not appear in the present calculations (Fig. 2b). For different wall velocities, wall shear distributions are presented in Fig. 3.

In Fig. 4, a typical example of streamline in case II for an upstream-moving wall is depicted. In case II, as in case I, the MRS separation point does not exist for upstream-moving walls, although it appears for downstream-moving walls. In Fig. 5, wall shear distributions are presented for different wall velocities. In Tsahalis' calculations, a point where the boundary-layer solution breaks down proceeds upstream with the increase in wall velocity ($-\alpha$, or AC in Tsahalis' notation) and is located upstream of a point of zero wall shear. Tsahalis thinks that at that point $\partial u/\partial y$ as well as u may vanish. In the present results, however, points where $\partial u/\partial y$ vanishes exist between two points of zero wall shear, as shown by the dashed line in Fig. 4. The author cannot imagine what physically reasonable lines of $\partial u/\partial y = 0$ and $u = 0$ meeting the MRS criterion can be drawn from Tsahalis' results. In Fig. 6, $\psi = 0$ lines and $u = 0$ lines are presented. In the present calculations, we could not determine a dividing streamline for the case of upstream-moving walls, because the results show only one stagnation point which apparently is a center of recirculating bubbles. However, for the case of upstream-moving walls, we can roughly estimate the scale of recirculating bubbles by plotting the $\psi = 0$ and $u = 0$ lines because these deviate more from the wall with the development of recirculating bubbles. With the increase in wall velocity ($|\alpha|$), recirculating bubbles

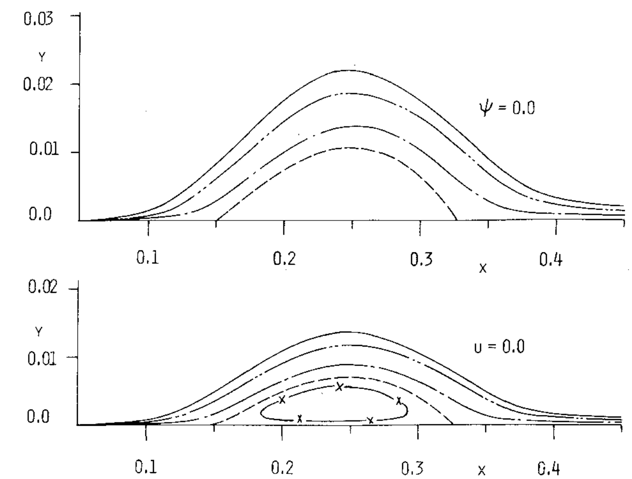


Fig. 6 $\psi = 0$ lines and $u = 0$ lines for case II: $Re = 9000$, $U_0 = 0.78$ (— $\alpha = -0.25$, ––––$\alpha = -0.15$, — — —$\alpha = -0.05$, ----$\alpha = 0.0$, —x—$\alpha = +0.025$).

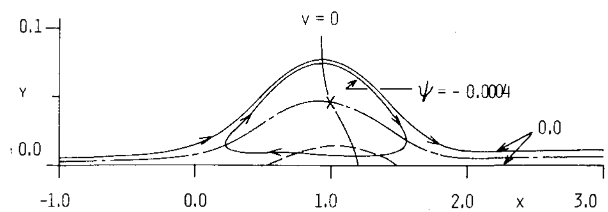


Fig. 7 Typical streamlines for case III: $Re = 6000$. $A = -0.08$, $\alpha = -0.05$, (X = stagnation point, — — —$u = 0$,---$\partial u / \partial y = 0$).

become larger for the case of upstream-moving walls and become smaller for the case of downstream-moving walls.

A typical example of streamlines for an upstream-moving wall in case III is shown in Fig. 7. In case III, as in other cases, the MRS criterion is never satisfied for upstream-moving walls.

In every case treated in the present paper, even in the case of the flow quite similar to that considered by Tsahalis, the MRS criterion is never satisfied for upstream-moving walls, although for downstream-moving walls it is satisfied. The similar conclusion has been obtained by Fansler and Danberg[12] from an integral boundary-layer approach. On the other hand, Koromilas and Telionis[7] experimentally obtained a dividing streamline around a rotating cylinder, which is similar to that expected by the MRS model. For such flows, the term $u''_w y$ in Eq. (4b), which vanishes in the present examples, may play an important role. Furthermore, for flows such as those around a rotating cylinder, the effects of wall curvature and the term $(\partial^2 \omega / \partial x^2)/Re$ may also be important. Studies of these effects are left for the future.

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